Aalto University
School of Electrical Engineering

Quantum Computation

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Important results in Quantum Computation

1985 David Deutsch: *Can a Quantum Computer efficiently solve problems that have no efficient solution on a classical computer?*

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1990s *Quantum Computers can efficiently simulate physical systems that can not be efficiently simulated on classical computers*

2009 Harrow, Hassidim, Lloyd: *Exponential speedup in solving linear equations*

Quantum bits

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- Similar to a classical bit, a qubit can take states $|0\rangle$ or $|1\rangle$

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Bloch-sphere representation of a Qubit

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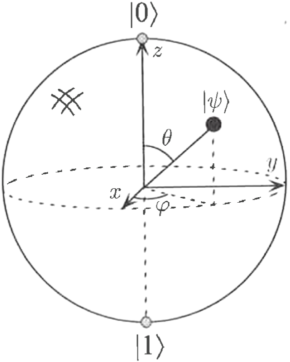
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

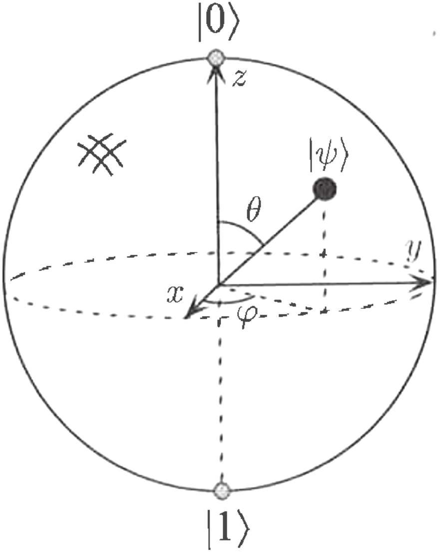
(Ignore $e^{i\gamma}$ since it has no observable effect)



Quantum bits

Measurement of qubits

- Infinite number of possible states for a single qubit.

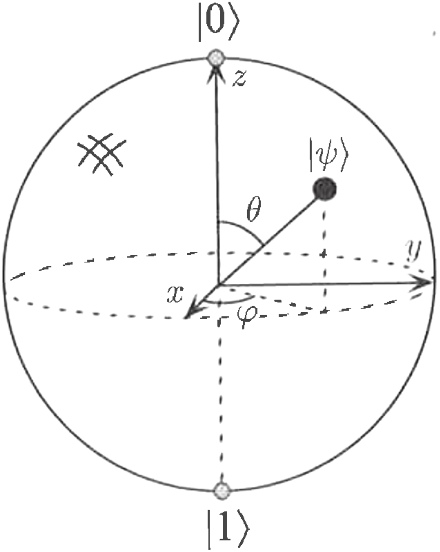


Quantum bits

Measurement of qubits

- Infinite number of possible states for a single qubit.
- However, measurement of $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ yields

  0 with probability $|\alpha|^2$
  1 with probability $|\beta|^2$



Multiple qubits

- Systems of multiple qubits are described accordingly:
  - A 2-qubit system has four computational basis states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ with

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$\sum_{i,j\in 0,1} |\alpha_{ij}|^2 = 1$$

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Remark A system of n qubits has computational basis states of the form $|x_1 x_2 \dots x_n\rangle$ and 2^n amplitudes. For larger n it becomes increasingly infeasible for a classical system to keep track of all individual amplitues

Introduction

Qubits

Quantum Computation

Quantum Algorithms

Quantum Computation

Single qubit gates

Quantum NOT-gate

$$X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{aligned} X|\psi\rangle &= X(\alpha|0\rangle + \beta|1\rangle) \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ &= \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \\ &= \beta|0\rangle + \alpha|1\rangle \end{aligned}$$

Requirements for quantum gates

Quantum gates need to be

- linear
  (*Violation of this rule would lead to paradoxes such as time travel and faster-than-light travel*)
- Unitary ($U^\dagger U = I$)
  (*In order to guarantee* $|\alpha|^2 + |\beta|^2 = 1$ *also after applying the transformation*)

Quantum Computation

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- Impossible: Copy qubits

Quantum Computation

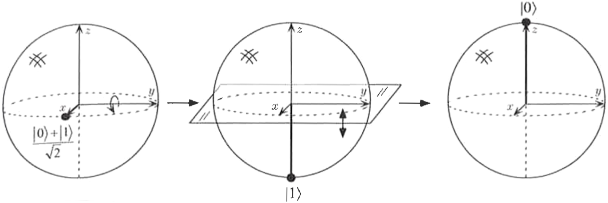
Single qubit gates

The Hadamard-gate

$$H \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The H-gate turns a $|0\rangle$ and $|1\rangle$ halfway between $|0\rangle$ and $|1\rangle$:

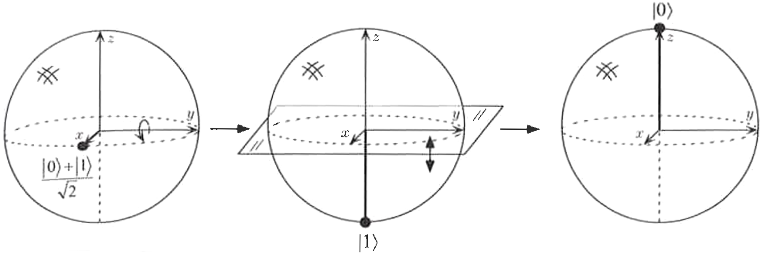
$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}; H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$



Quantum Computation

Decomposing single qubit operations

$$U = e^{i\alpha} \overbrace{\begin{bmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{bmatrix}}^{\text{rotation about z}} \overbrace{\begin{bmatrix} \cos\frac{\gamma}{1} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{bmatrix}}^{\text{rotation}}, \begin{bmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{bmatrix}$$



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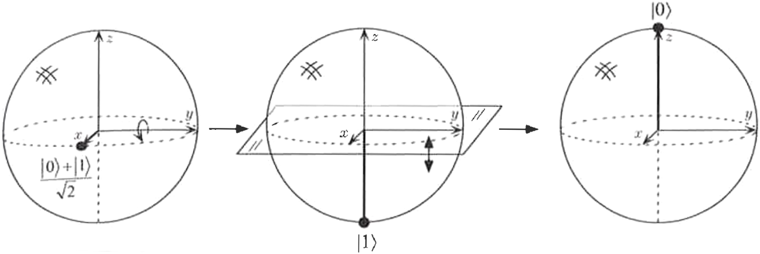


- Any Classical cirquit can be simulated with qubit gates

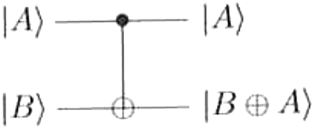
Quantum Computation

Multiple qubit gates

Controlled-NOT (CNOT)

Flip second qubit IFF control qubit is $|1\rangle$:

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

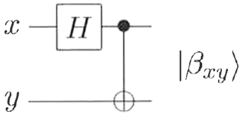


$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Quantum Computation

Bell states

In	Out
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2} \equiv \|\beta_{00}\rangle$
$\|01\rangle$	$(\|01\rangle + \|10\rangle)/\sqrt{2} \equiv \|\beta_{01}\rangle$
$\|10\rangle$	$(\|00\rangle - \|11\rangle)/\sqrt{2} \equiv \|\beta_{10}\rangle$
$\|11\rangle$	$(\|01\rangle - \|10\rangle)/\sqrt{2} \equiv \|\beta_{11}\rangle$



EPR states

Hadamard gate to put the top qubit in a superposition

CNOT gate superposition acts as control input to CNOT

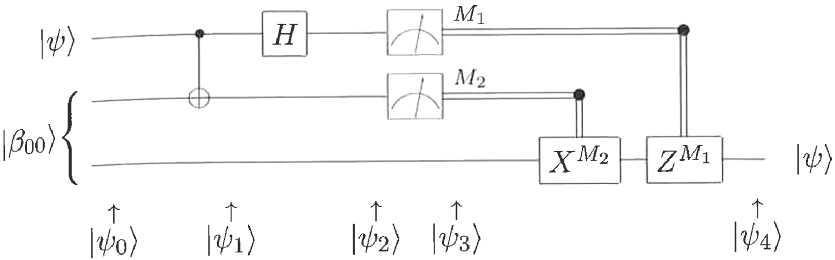
Example: $|00\rangle \xrightarrow{H} \frac{(|0\rangle+|1\rangle)|0\rangle}{\sqrt{2}} \xrightarrow{CNOT} \frac{|00\rangle+|11\rangle}{\sqrt{2}}$

Quantum teleportation

Quantum teleportation is a technique for moving quantum states around – even in the absense of a quantum communications channel

Quantum Computation

Quantum teleportation



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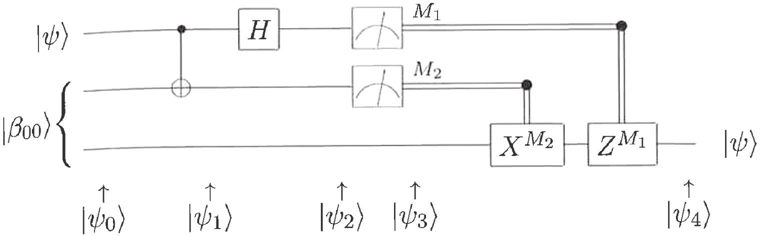
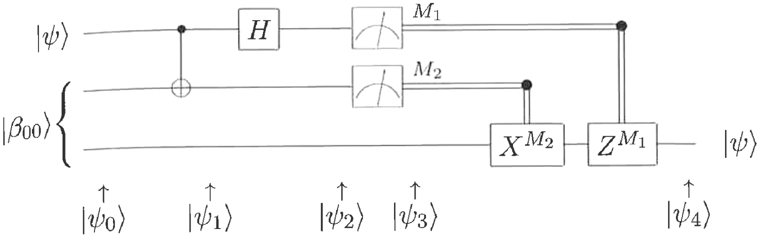
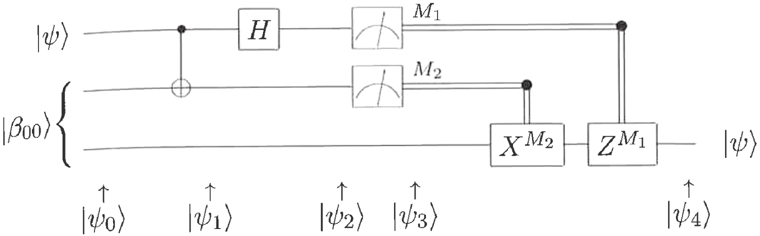


$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \\
|\psi_0\rangle &= |\psi\rangle|\beta_{00}\rangle \\
&= 1/\sqrt{2}\,[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)]
\end{aligned}$$



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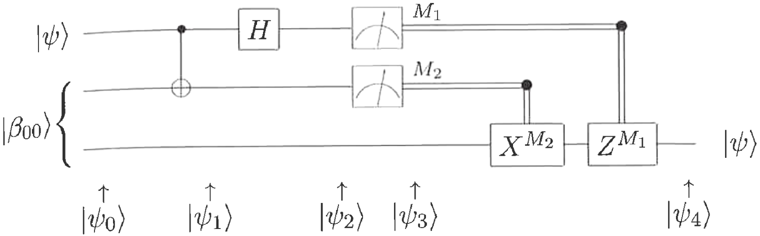

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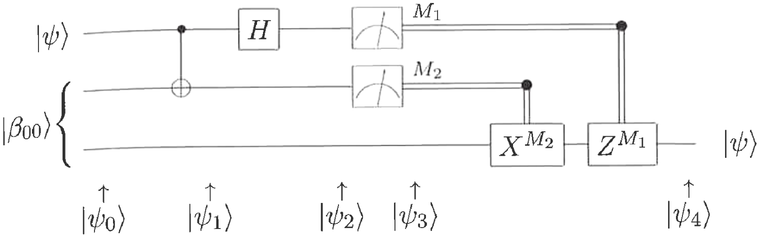

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&= 1/2\,[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\
&\quad + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)]
\end{aligned}
$$

Introduction

Qubits

Quantum Computation

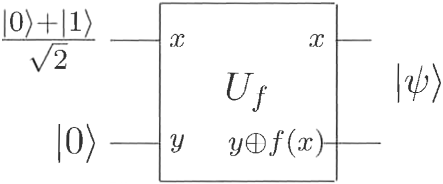
Quantum Algorithms

Quantum algorithms

Quantum parallelism

Evaluating a function for multiple inputs simultaneously

- $f(x) : \{0,1\} \rightarrow \{0,1\}$
- $\oplus$: addition modulo 2



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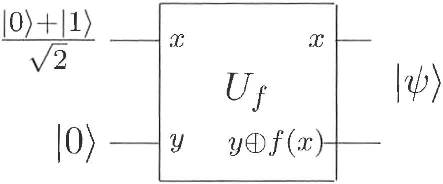
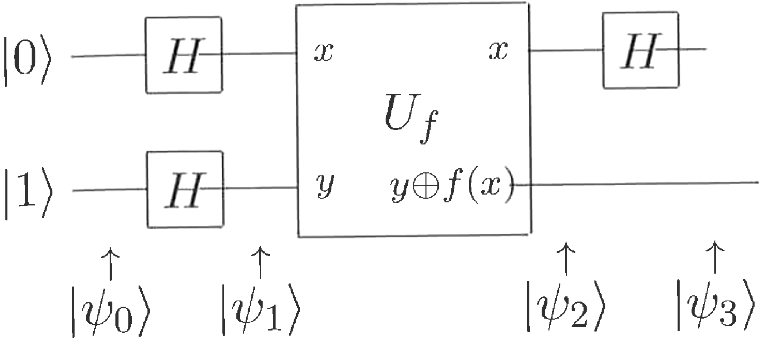


- Applying U_f results in

$$\frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

Quantum algorithms

Deutsch's algorithm



Quantum algorithms

Deutsch's algorithm

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Quantum algorithms

Deutsch's algorithm

- $|\psi_0\rangle = |01\rangle$
- $|\psi_1\rangle = \left[\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$

Quantum algorithms

Deutsch's algorithm

If we apply U_f to $\frac{|x\rangle(|0\rangle-|1\rangle)}{\sqrt{2}}$ we obtain $(-1)^{f(x)}\frac{|x\rangle(|0\rangle-|1\rangle)}{\sqrt{2}}$

- $|\psi_0\rangle = |01\rangle$
- $|\psi_1\rangle = \left[\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$
- $|\psi_2\rangle = \begin{cases} \pm\left[\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] & \text{if } f(0)=f(1) \\ \pm\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] & \text{if } f(0)\neq f(1) \end{cases}$

Quantum algorithms

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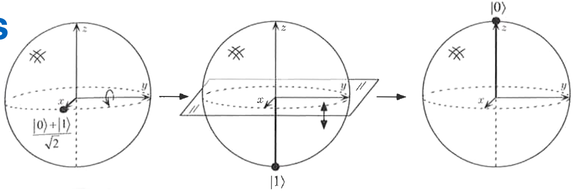

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Quantum algorithms

Deutsch's algorithm

Since
$f(0) \oplus f(1) = 0$
IFF $f(0) = f(1)$

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Quantum algorithms

Deutsch's algorithm

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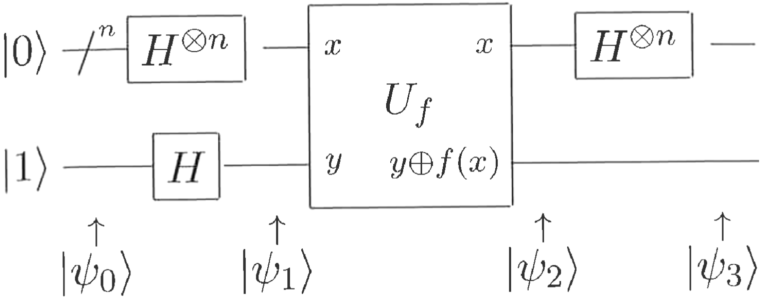
- $= \pm|f(0) \oplus f(1)\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$
- Measuring the first qubit determines a global property with just a single evaluation of $f(x)$

Quantum algorithms – outlook

Deutsch-Jozsa algorithm



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Thank you!

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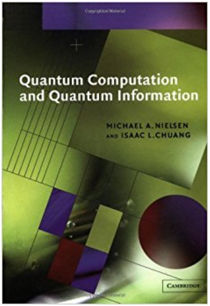
Literature

- M.A. Nielsen and I.L. Chuang: Quantum Computation and Quantum Information, Cambridge, 2000.



Further reading

- M.A. Nielsen and I.L. Chuang: Quantum Computation and Quantum Information, Cambridge, 2000.
- A. Montanaro: Quantum algorithms: an overview, Npj Quantum Information, (2) 2016
- Childs, van Dam: Quantum algorithms for algebraic problems, Rev. Mod. Phys. 82, 2010
- M. Mosca: Computational Complexity, 2303-2333, Springer, 2012
- M. Santha: Quantum walk based search algorithms, Theory Appl. Model. Comput. 4978, 31-46, 2008
- D. Bacon, W. van Dam: Recent progress in quantum algorithms, Commun. ACM 53, 84-93, 2010

Links

Quantum Algorithm Zoo http://math.nist.gov/quantum/zoo/

Timeline of Quantum Computation https://en.wikipedia.org /wiki/Timeline_of_quantum_computing

Quantum Computation
Single qubit gates

The Z-gate

$$Z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The Z gate leaves $|0\rangle$ unchanged and flips $|1\rangle$ to $-|1\rangle$

Quantum Computation

Copying qubits

Is is possible to copy qubits?

- In classical cirquits, it is possible to copy bits with the help of a CNOT gate
- Can we use the CNOT qubit gate to copy qubits?

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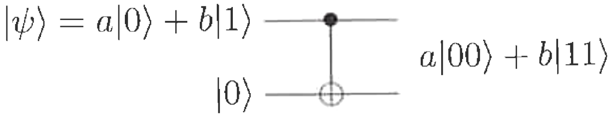

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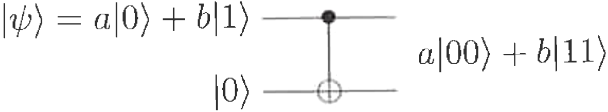
- Did we copy $|\psi\rangle$?



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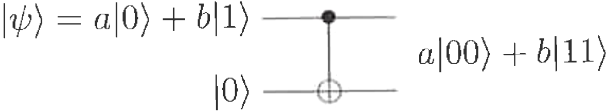
- Did we copy $|\psi\rangle$?
- Show equation 1.22



Quantum Computation

Copying of qubits

No-cloning theorem

Aim $|\psi\rangle \otimes |s\rangle \xrightarrow{U} U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle$

Quantum Computation

Copying of qubits

No-cloning theorem

Aim $|\psi\rangle \otimes |s\rangle \xrightarrow{U} U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle$

Assume

$$U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle$$
$$U(|\varphi\rangle \otimes |s\rangle) = |\varphi\rangle \otimes |\varphi\rangle$$

Quantum Computation

Copying of qubits

No-cloning theorem

Aim $|\psi\rangle \otimes |s\rangle \xrightarrow{U} U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle$

Assume

$$U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle$$
$$U(|\varphi\rangle \otimes |s\rangle) = |\varphi\rangle \otimes |\varphi\rangle$$

Contradiction The inner product of these to equations gives

$$\langle\psi|\varphi\rangle = (\langle\psi|\varphi\rangle)^2$$

$x = x^2$ is only possible for $x \in \{0, 1\}$

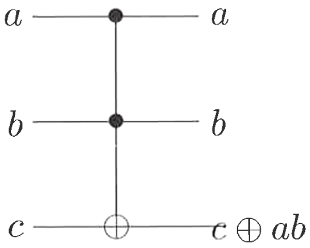
Quantum algorithms

Classical computations on a quantum computer

Simulation of classical cirquits

Any classical circuit can be replaced by an equivalent quantum circuit containing only reversible elements by using Toffoli gates

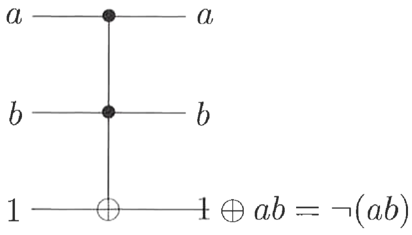
Inputs			Outputs		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



Quantum algorithms

Classical computations on a quantum computer

NAND



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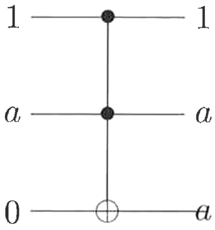


Quantum Computation

qubit gates

Restrictions for multiple qubit gates

Most classical gates not directly convertable to qubit gates since they are non-invertible and irreversible (Unitary requirement).

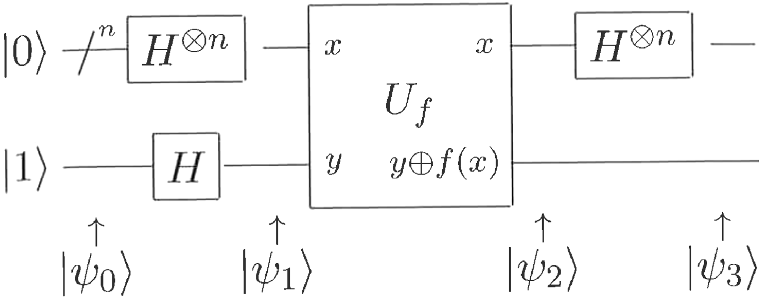
Unitary quantum gates are always invertible

Universality result

Any multiple qubit logic gate may be composed from CNOT and single qubit gates

Quantum algorithms

Deutsch-Jozsa algorithm



Quantum algorithms

Deutsch-Jozsa algorithm

- $|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$

Quantum algorithms

Deutsch-Jozsa algorithm

- $|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$
- $|\psi_1\rangle = \sum_{x\in\{0,1\}^n} \frac{|x\rangle}{\sqrt{2^n}} \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$

Quantum algorithms

Deutsch-Jozsa algorithm

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- $|\psi_2\rangle = \sum_{x} \frac{(-1)^{f(x)}|x\rangle}{\sqrt{2^n}} \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$

Quantum algorithms

Deutsch-Jozsa algorithm

- $|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$
- $|\psi_1\rangle = \sum_{x\in\{0,1\}^n} \frac{|x\rangle}{\sqrt{2^n}} \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$
- $|\psi_2\rangle = \sum_x \frac{(-1)^{f(x)}|x\rangle}{\sqrt{2^n}} \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$
- $|\psi_3\rangle = \sum_z \sum_x \frac{(-1)^{x\cdot z+f(x)}|z\rangle}{2^n} \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$